All about Value at Risk (VaR)

By: Peter Urbani

Value at Risk (VaR) is slowly replacing standard deviation or volatility as the most widely used measure of risk.

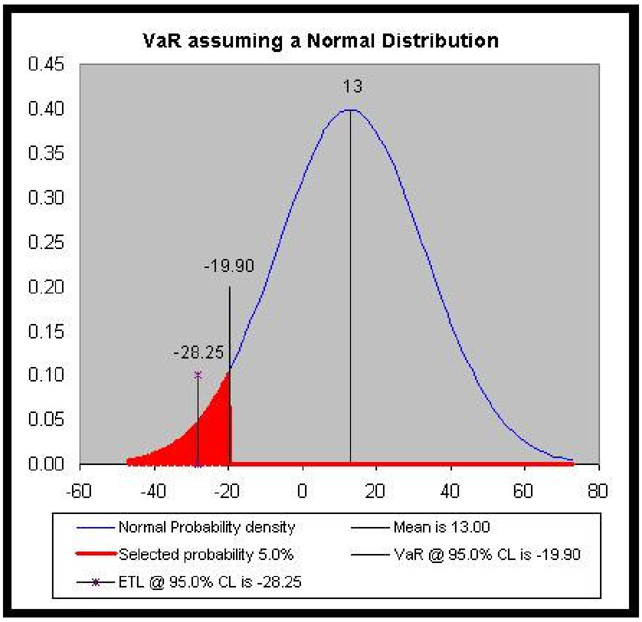
This has come about because of the need for a single risk measure for the setting of capital adequacy limits for banks and other financial institutions. In 1993 the Bank of International Settlements (BIS) members met in Basle and amended the Basle Accord to require Banks and other financial institutions to hold in reserve enough capital to cover 10 days of potential losses based on the 95% 10-day VaR. To ensure that this measure was sufficiently conservative it was also multiplied by a scaling factor ranging from 1 - 3 times. Financial institutions, including SA banks, were furthermore required to report their overall risk exposure on this basis. The resulting Basle I Accord came into force in 1998 and the subsequent Basle II accord which was implemented in 2003 added the requirement to also quantify operational risks by 2008.

What Value at Risk allows is for regulators and bank presidents to put a single number on their worst-case scenario and to plan for it accordingly. The appeal of such a single risk number is obvious but, as with all one-size fits-all measures, potentially misleading for the uninformed.

Value at Risk is a percentile-based risk-measure that measures the expected loss of a portfolio over a specified period of time for a set level of probability or confidence.

For example a portfolio of R1m with an average expected annual return of 13% and a standard deviation of 20% would have a 10 day VaR of R 60,370 at the 95th percentile. This means that your losses over any 10-day period should only exceed R 60,370, 5% of the time or roughly 1 every year.

Some text-books describe VaR as the maximum loss you may experience. This is technically correct but only for the given confidence level. There is always a higher level of loss for a higher confidence level. The average expected loss greater than a given confidence level is called the conditional value at risk (CVaR) aka. Expected Shortfall and Extreme Tail Loss.



	Years	Months	Weeks	Days
Mean	13	1.08	0.25	0.05
SD	20	5.77	2.77	1.26
CL	0.95	0.95	0.95	0.95
HPR	1	12	52	**10**
Z-SCORE	-1.645	-1.645	-1.645	-1.645
VaR	-19.90	-19.90	-19.90	-6.04

=Mean*HPR+((NORMSINV(1-CL))*SD*SQRT(HPR))

There are three main methods of calculating Value at Risk, they are:

1.) The Historical or empirical method

2.) The Parametric or analytic method

3.) The Simulation or Monte-Carlo method

The Historical method involves simply taking the empirical P/L history and ordering it. Suppose we have 100 observations of the returns of our portfolio. Using a spreadsheet we would simply order the returns from largest to smallest. The Value at Risk for the 95th percentile would then be the 6th largest loss.

The advantage of the Historical method is that it requires no assumption to be made about the nature or shape of the distribution of returns. The disadvantage is that we are thus implicitly assuming that the shape of future returns will be the same as those of the past.

For this to be statistically likely we need to ensure that we have a sufficient number of observations and that they are representative of all possible states of the portfolio (i.e. incorporate data from both bull and bear markets). Using monthly data statistical certainty would typically require 34 years of data before we could be comfortable with this assumption. Since we seldom if ever have this much history the empirical method is not considered as accurate as either the parametric or simulation method.

The parametric or analytic method requires an assumption to be made about the statistical distribution (normal, log-normal etc.) from which the data is drawn. We can think of parametric approaches as fitting curves through the data and then reading off the VaR from the fitted curve. The attraction of parametric VaR is that relatively little information is needed to compute it.. The main weakness is that the distribution chosen may not accurately reflect all possible states of the market and may under or overestimate the risk. This problem is particularly acute when using value at risk to assess the risk of asymmetric distributions such as portfolios containing options and hedge funds. In such cases the higher statistical moments of skewness and kurtosis which contribute to more extreme losses ('Fat tails') need to be taken into account. Fortunately closed form formulas now exist for distributions such as the Student T, the Extreme Value or Generalised Pareto distribution and for modifying the normal VaR to take account of excess skewness and kurtosis using the Cornish-Fisher expansion. The Forsey-Sortino 3-parameter log-normal distribution also gives a better fit to most financial time series.

So although some level of statistical sophistication is necessary, parametric methods exist for a wide variety of distributions.

The general form for calculating parametric VaR is:

Mean x HPR + (Z-Score x Std Dev x SQRT(HPR)

Where:

Mean = Average expected return
Std Dev = Standard deviation
HPR = Holding period
Z-Score=probability

The simulation or Monte-Carlo method of calculating VaR has become increasingly popular in recent years due to the dramatic increase in the availability and power of desktop PC's. As the name implies simulation VaR generates many thousand simulated returns drawn either from a parametric assumption about the shape of the distribution or preferably by re-sampling the empirical history and generating enough data to be statistically significant and then ordering them and reading off the desired percentile as in the historic calculation method.

Different VaR Calculations

P/L					
50.95	Mean	13			
-32.56	SD	20			
-8.93	Skew	0.13			
-22.22	Kurt	0.00			
17.82	CL	0.95			
9.78	HPR	1			
20.50					
-2.12	**HISTORIC**				
23.26	VaR HS	-19.56			
59.34	ETL HS	-27.91			
-18.07					
1.67	**SIMULATION**				
11.82	VaR Bootstrap	-19.56			
-0.46	VaR MC	-19.62			
48.43	ETL Bootstrap	-27.23			
57.93					
11.96	**PARAMETRIC**	Normal	Student T	CF VaR	Gumbel
49.00	VaR	-19.90	-19.89	-20.50	-24.32
-32.50	ETL	-28.25	-28.25	-18.24	-39.98
14.69					
-33.34	**BACKTEST**				
-27.03	Obs less than VaR	25			
31.14	Percentage	0.05			
21.13	Kupiec Test	0.55			
6.46	Pass/Fail	Pass			
-36.83					
28.45					
49.01					

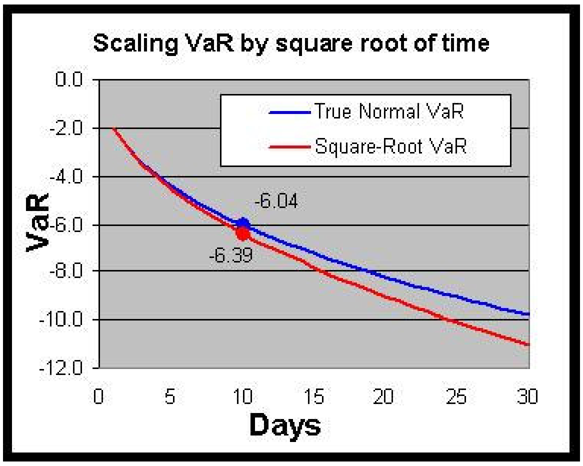
All three methods have their place with the Monte-Carlo simulation re-sampling or bootstrap method probably being the best for the uninformed. Particular care should be taken not to use the normal VaR calculation where the distribution is know- or suspected to-be asymmetric as in the case of Hedge funds and emerging markets. Unfortunately this is seldom done.

Another problem is the assumption by some calculations of a zero mean. Whilst this is generally appropriate for daily VaR calculations (The expected daily return of most markets being statistically insignificantly different from 0) it is certainly not the case for longer holding periods. The common practice of scaling VaR calculations by the square root of time rule can also lead to serious over or under-estimation of the actual value at risk

It is fairly common to convert daily standard deviation into Monthly or Annual standard deviation by multiplying or diving by the square root of time as necessary. Thus converting the Annual standard deviation of 20 in the example shown to a daily number is given by 20/SQRT(252) = 1.26

This is fine for the standard deviation, but if we calculate the 1-day VaR correctly as -2,02% and then divide it by the square root of 10 to get the 10-Day VaR we get -6,39% which is wrong. The correct number is only -6,04%. Whilst this error may seem small, after 30 days scaling by the square root of time will over-estimate the true VaR by almost 13%.

Thus care should always be taken to calculate VaR correctly and to use the same frequency of data.



The final problem with Value at Risk is perhaps the most serious namely that it may not be the 'best' available measure of risk in that, according to *Atzner et Al,* it is not a mathematically fully coherent measure. Without getting too technical this is because, excepting in the special case of the normal distribution, VaR does not satisfy the sub-additivity requirement for mathematical coherence. In other words the sum of your component VaR's may be greater or less than that of the whole except in the aforementioned special case.

Apart from the obvious confusion this may cause it also results in the Mean / Value at Risk frontier of investments not always being a smooth convex shape. This can cause problems when trying to find the optimal point as the gradient point search methods used in most solver engines can get trapped in a local minima or maxima and not always find the global maxima or minima you are looking for. Fortunately the solution is simply to use the conditional Value at Risk which is fully mathematically coherent (see Uryasev).

I Hope I have given you a reasonable idea of the uses and misuses of Value at Risk. Used correctly it can be an invaluable tool in putting a single number on the potential losses which might occur. However given number of different calculation methods and widespread presence of 'fat tail's' in financial time series great care should be taken to use the right measure.

As with all quantitative measures it is also advisable to use experience and common sense. No amount of bootstrapping or simulating a small sample of data such as the returns on the Nasdaq from 1998 – 2000 will open your eyes to the true potential for loss. Any sample period which is particularly short or which has an abnormally low standard deviation should be looked at twice. It is also a good idea to stress test VaR calculations by looking at historic worst case scenarios and using the correlation matrices from such periods to stress test your assumptions.

References:

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A History of Value at Risk – Glyn A Holton (2002)

Mastering Value at Risk – Cormac Butler (1999)

Value at Risk – Phillipe Jorion (1997)

Managing Downside risk – Sortino&Satchell (2001)

Coherent Measures of Risk – Artzner (1999)

Conditional Value-at-Risk – Uryasev (2001)

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